Implementation of interaction diagrams in the finite element open source software XC

Luis C. Pérez Tato[1], Ana Ortega[2]

Abstract
XC has taken from OpenSees the implementation of fiber models for the analysis of cross-sections under normal stresses. This is the backbone around which the program carries out the structural verification according to different standards. In this paper we describe the procedure followed to implement in XC the generation of 3D interaction diagrams for reinforced concrete sections. A more general, but also more time-consuming process and not so straight-converging analysis is outlined for generating the interaction diagram of generic sections made up of any materials combination. A practical example is provided in the last section of the document.

[1] *structural engineer - l.pereztato@gmail.com*
[2] *structural engineer - ana.Ortega.Ort@gmail.con*

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1. Fiber models to discretize the cross-sections

XC takes from OpenSees the fiber models to simulate the cross-sections behaviour. This is the backbone around which XC carries out the structural verification according to different standards (EC2, EHE, SIA, ...).

The analysis of cross-sections by means of fiber models is based on their discretization in a series of point elements or fibers. Each fiber is characterized by the location of its center of gravity, a tributary area and a non-linear constitutive model, as shown in figure 1. Uniaxial strain at each fiber is computed from the generalized strains by means of the plane-section hypothesis.

In this regard, fiber-like sectional analysis offers several advantages:

1. Its large capacity and versatility for reproducing complex geometry and materials combinations makes it appropriate to deal with arbitrary shaped sections and any material behavior.
2. Fiber stress-strain relations are typically expressed as explicit functions of strain, not needing to split them in polynomial segments, cumbersome to deal with.
3. It is adequate to perform nonlinear inelastic analysis; the nonlinear behavior of the elements derives entirely from the nonlinear stress-strain relation of the materials.
4. It has been proved to produce consistent results compared to experimental results [4].

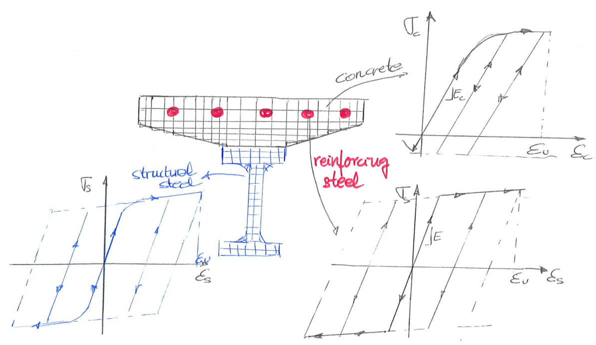

Figure 1. Fiber model of a cross section made up of various materials

On the other hand, the accuracy of the response is affected by the number of fibers in the section. A large number of fibers certainly gives better results, but computational cost increases with it, since the history variables necessary to track the hysteretic behavior of each fiber must be stored at each iteration.

2. Generation of points for the interaction diagram

The generation of 3D interaction diagrams for reinforced concrete sections is implemented in XC by following the procedure that is described in this section. The main assumption is that plane sections remain plane (and normal to the reference axis during the deformation history). This leads to a simple geometric relation between section generalized deformations and fiber strains.

The following steps are taken (see figure 2):

1. The cross-section is discretized in fibers. The geometric characteristics of a fiber are its location in the local *y,z* reference system and the fiber area A_{jk}; a material constitutive model is assigned to each fiber.
2. A direction of the neutral axis is assumed (defined by the angle θ), and a working coordinates system Y'Z' chosen, such that the Y' axis is parallel to the direction of the neutral axis, so that the strains and stresses vary only in Z' axis.
3. For each value of the bending axis direction (angle θ):
   (a) The outermost tensioned steel fiber [1] and the outermost compressed concrete fiber in the cross-section are found.
   (b) A succession of specific planes of deformation are imposed to the cross-section in order to obtain some representative points of the 2D interaction diagram for the chosen direction of the neutral axis. Those planes of deformation (figure 2) are the following :
      i. Vertical plane of deformation (χ= -∞) in pure tension, corresponding to an elongation in the reinforcement of 10‰.
      ii. Limit plane in combined tension (χ= 0) obtained by turning the previous plane about the axis A.
      iii. Plane of deformation in combined bending, obtained by turning the previous plane about the axis A until the strain in the most compressed fiber of concrete equals to its ultimate strain ε_{cu}.
      iv. Plane of deformation in combined bending, defined by an strain in the most compressed fiber of concrete equal to its ultimate strain ε_{cu} and an elongation of the most tensioned reinforcement corresponding to the yield stress of the steel ε_y.
      v. Plane of deformation (χ= d) where all the fibers are compressed and the strains range from 0 to the ultimate strain of concrete ε_{cu}.
      vi. Vertical plane of deformation (χ= ∞) in pure compression, corresponding to a strain equal to the maximum compressive strain in the concrete under simple compression ε_{c0}.
   (c) For each plane of deformation imposed, the uniaxial stress of the fibers are integrated to obtain the axial force N and bending moments M_Y and M_Z at the failure point.
4. This procedure is repeated for different θ directions of the neutral axis, allowing the generation of multiple points that will be integrated in the 3D interaction surfaceby means of a convex hull algorithm, as described in section 3.

[1] For now, steel rebars and concrete are the only materials considered. The introduction of materials different from those may require the modification of the pivots diagram shown in figure 2

An alternative approach

A more general, but also more time-consuming process and not so straight-converging analysis for generating the interaction diagram of sections could be developed in the following steps:

1. After discretizing the cross-section in fibers, the following process is followed for each value of the axial force:
   (a) A direction of the neutral axis is assumed (defined by the angle θ), and we chose a working coordinates system Y'Z' such that the Y' axis is parallel to the direction of the neutral axis, so that the strains and stresses vary only in Z' axis.
   (b) A curvature k is initialized and the placement of the neutral axis determined by applying constitutive and equilibrium conditions; small increments of Δk are applied as imposed curvature. This incremental procedure continues until one or the materials reaches the ultimate compressive or tensile strain. The integration of the uniaxial stress of the fibers allows to calculate the bending moments M_Y and M_Z at the failure point.
   (c) The interaction curve for a given axial load is constructed, as shown in fig. 4, by repeating this procedure for different θ directions of the neutral axis.
2. The 3D interaction surface of the cross-section is constructed by integrating the interaction curves calculated for multiple axial loads.

This alternative approach doesn't make use of a pivots diagram and, this way, you can use any materials in the fiber model.

3. Convex Hull, assembling the diagram

To construct the 3D interaction surface of the cross-section draw from the sets of internal forces (N, M_y, M_z) calculated as previously stated, the diagram is assumed to be convex [2], so that a convex hull algorithm can be used for this purpose.

In mathematics, the convex hull [5] or convex envelope of a set X of points in the Euclidean plane or Euclidean space is the smallest convex set that contains X; for instance, when X is a bounded subset of a plane, the convex hull may be

[2] Apparently, this contention is only correct if the stress-strain law of the material is a non-decreasing function of strain

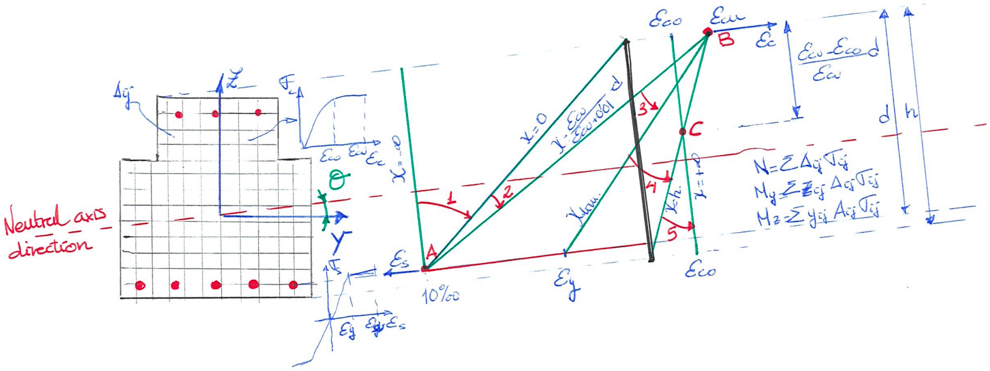

Figure 2. Process to obtain a set of points of the interaction diagram for a RC section

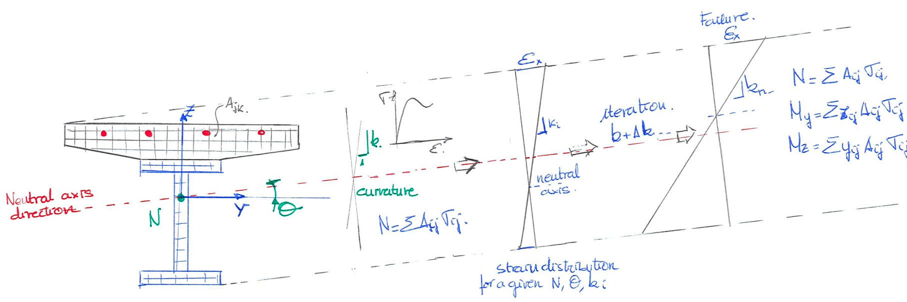

Figure 3. Generic process to obtain one point of the interaction diagram

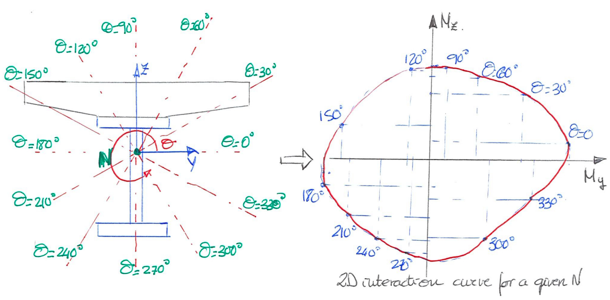

Figure 4. Process to obtain a failure curve for a certain value of the axial load

visualized as the shape enclosed by a rubber band streched around X.

Under the project *Computational Geometry Algorithms Library* (CGAL) a function has been developed to compute the convex hull of a given set of three-dimensional points. This is the algorithm that we have incoporated to the open source FE program **XC** to construct the 3D interaction diagrams, which is the subject of this article.

4. Examples

The **XC** source-code is hosted in the platform GitHub, at:

XC source-code.

There you will find some verification tests related to the generation of interaction diagrams:

test_diag_interaccion01.py
test_diag_interaccion02.py
test_diag_interaccion03.py
test_diag_interaccion04.py
test_diag_interaccion05.py
test_diag_interaccion06.py

To generate the diagram in figure 5, you can use the following script:

interaction_diagram_mayavi.py

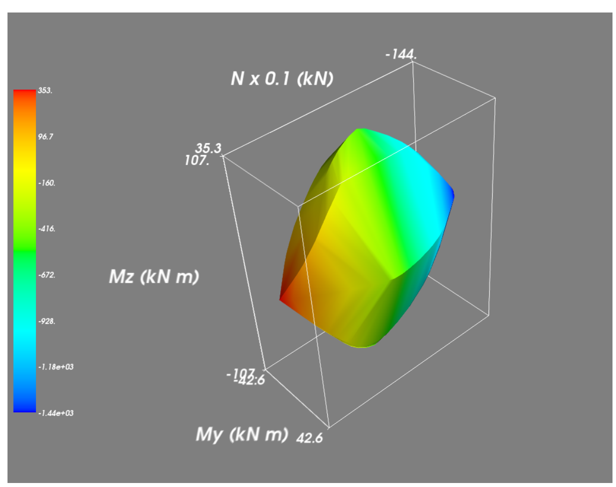

Figure 5. Example of interaction diagram generated by XC

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[5] Wikipedia. Convex hull.